

# Driven Disk Mechanical Resonator Properties (MKS units unless otherwise noted)

Last Modified 00/09/10

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This is the outline of the transducer mechanical coupling through the disk-post resonator. See "Stimulate Condensation Resonator.xmcd" from the home pages for the third sound treatment from which this was derived.

The modes are assumed to be those of the free disk, but corrections are included for the non-zero disk width. See "disk post plane.xmcd" and "free disk.xmcd" for a comparison of the actual modes.

## Contents

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### Physical Properties

$$\begin{aligned} \epsilon_0 &:= 8.85 \cdot 10^{-12} & k &:= 1.3805 \cdot 10^{-23} \\ \text{sapphire} & & Y &:= 4 \cdot 10^{11} & \nu &:= 0.29 & \rho &:= 3980 & a &:= 0.0065 & h &:= 0.0005 \\ \epsilon_{\text{He}} &:= 1.055 & \epsilon_{\text{N}_2} &:= 1.438 & & & & & & & & \text{for calibration purposes} \end{aligned}$$

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### Physical Properties

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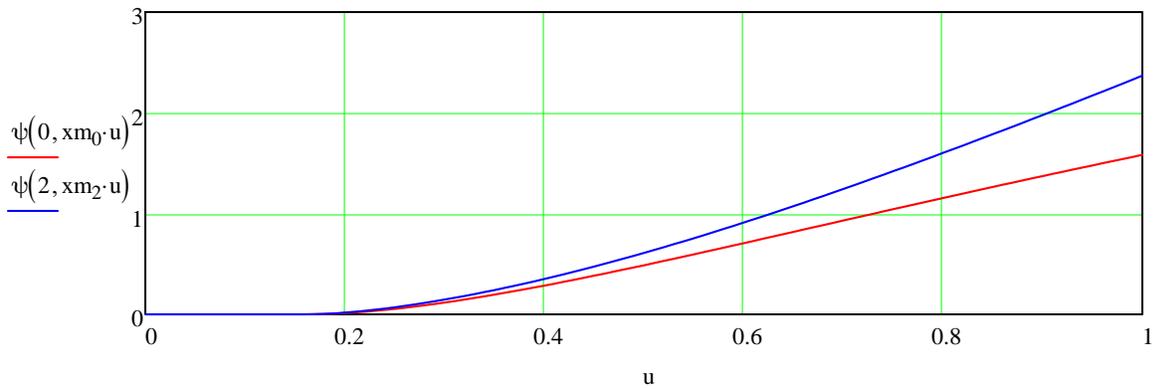
### Disk Displacements

Numerical values from "annular plate vibrations.xmcd" for m=0 and m=2 modes...

$$\epsilon := 0.16 \quad x_{m_0} := 2.17749 \quad x_{m_2} := 2.46491$$

$$\psi(m, x) := \begin{cases} 0 & \text{if } x \leq \varepsilon \cdot x_{m0} \\ \text{otherwise} & \begin{cases} -0.82571 \cdot J_n(m, x) + 0.32409 \cdot I_n(m, x) + 1.41689 \cdot Y_n(m, x) + 1.18341 \cdot K_n(m, x) & \text{if } m = 0 \\ 3.64573 \cdot J_n(m, x) + 0.68742 \cdot I_n(m, x) + 0.25763 \cdot Y_n(m, x) + 0.17049 \cdot K_n(m, x) & \text{if } m = 2 \end{cases} \end{cases}$$

u := 0, .01 .. 1



$$\eta(r, \varphi) = \eta_0 \cdot \psi(k \cdot r) \cdot \cos(m \cdot \varphi) \quad \frac{1}{\pi \cdot a^2} \cdot \int_0^a \int_0^{2 \cdot \pi} \eta(r, \varphi)^2 \cdot r \, d\varphi \, dr = 1$$

$$k = \frac{x_0}{a} \quad \omega = h \cdot \left( \frac{x_0}{a} \right)^2 \cdot \sqrt{\frac{Y}{12 \cdot \rho \cdot (1 - \nu^2)}}$$

$$\text{peak kinetic energy} \quad KE = \int \frac{1}{2} \cdot \rho \cdot h \cdot (\omega \cdot \eta(x, \varphi))^2 \, dA = \frac{1}{2} \cdot \rho \cdot \pi \cdot a^2 \cdot h \cdot (\omega \cdot \eta_0)^2$$

peak KE = peak elastic energy = total energy (standing wave modes)

Peak elastic energy in terms of the elastic properties

$$U_{\text{elastic}} = \frac{1}{24} \cdot \pi \cdot \frac{Y \cdot a^2 \cdot h^3 \cdot k^4}{(1 - \nu^2)} \cdot \eta_0^2$$

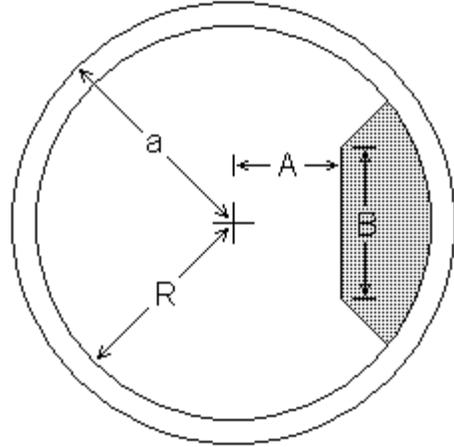
Disk Displacements

Cell Properties

See "/stimulated condensation\pickup electrode shape.xmcd" for the original derivation.

The drive or pickup electrodes are assumed to be parallel plate gaps, neglecting any fringe fields. Each electrode is characterized by its area and a quantity (D or P below) reflecting its spatial overlap with the mode of interest. The active region of the electrodes are determined by the overlapping regions of the disk (at ground potential) and the electrode plates (applied or circuit potential).

The disk outer radius (a) and the capacitor outer radius (R) are different: The capacitor gap increases near the outer radius of the disk due to the deviation from flatness of the polished surface. (note that for the third sound modes, the disk radius a is also artificially increased to account for the wave overlapping the outer edge).



actual disk outer radius  $a_0 := 0.0065$

disk thickness  $d := 0.0005$

effective flat disk radius ( $a_0 + \frac{1}{2} \cdot d$  for third sound)  $a := a_0$

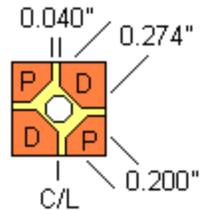
fraction of actual radius forming the small gap  $x_{gap} := 0.92$   $a = 6.5 \times 10^{-3}$

scaled capacitor outer radius  $R := \frac{x_{gap} \cdot a_0}{a}$   $R = 0.92$

The "post cell" electrode geometry can be defined by two additional dimensions: The inner edge distance from the center (A) and the length of this inner edge (B). The outer edge is at the effective capacitor radius (R) and the sides extend at 45° between the inner edge and outer radius, all scaled to the disk radius a.

drive  $A := \frac{0.200 - 0.0254}{2 \cdot a}$   $B := 2 \cdot A$

pickup  $A := \frac{0.274 - 0.0254}{2 \cdot a}$   $B := 2 \cdot \left( A - \sqrt{2} \cdot \frac{0.040 - 0.0254}{a} \right)$



$$A_p := \begin{cases} C \leftarrow \frac{2 \cdot A - B + \sqrt{8 \cdot R^2 - (2 \cdot A - B)^2}}{4} \\ c \leftarrow \frac{C}{R} \\ 2 \cdot a^2 \cdot \left[ (C - A) \cdot (B + C - A) + R^2 \cdot \left( \arccos(c) - c \cdot \sqrt{1 - c^2} \right) \right] \end{cases}$$

C is the scaled x value at the intersections of the 45° lines and the circle at R

$$A_p = 2.57 \times 10^{-5} \quad \text{square meters, both pickup plates in parallel}$$

### Drive and Pickup Integrals

These are the geometrical overlaps of the wave mode with the electrodes. For convenience, they are normalized to the full circle. Their use is derived in the "Mode Details" section..

$$D, P = \frac{1}{\pi \cdot a^2} \int \psi(m, k \cdot r) \cdot e^{i \cdot m \cdot \phi} dA \quad \text{integrate over the regions exposed to the uniform gap of the parallel plate}$$

pickup plate parameters

$$\underline{A} := \frac{0.274 \cdot 0.0254}{2 \cdot a} \quad \underline{B} := 2 \cdot \left( A - \sqrt{2} \cdot \frac{0.040 \cdot 0.0254}{a} \right)$$

$$\begin{aligned}
P(m) := & \left| \begin{array}{l}
x0m \leftarrow xm_m \\
rc \leftarrow \sqrt{A^2 + \left(\frac{B}{2}\right)^2} \\
\text{if } m > 0 \\
\left| \begin{array}{l}
p1 \leftarrow \int_A^{rc} \psi(m, x0m \cdot r) \cdot \sin\left(m \cdot \text{acos}\left(\frac{A}{r}\right)\right) \cdot r \, dr \\
p2 \leftarrow \int_{rc}^R \psi(m, x0m \cdot r) \cdot \sin\left[m \cdot \left(\frac{\pi}{4} - \text{asin}\left(\frac{A - \frac{B}{2}}{\sqrt{2} \cdot r}\right)\right)\right] \cdot r \, dr \\
\text{if } \left[\text{mod}(m, 2) = 0, \frac{4}{m \cdot \pi} \cdot (p1 + p2), 0\right] \\
\text{otherwise} \\
\left| \begin{array}{l}
p1 \leftarrow \int_A^{rc} \psi(m, x0m \cdot r) \cdot \text{acos}\left(\frac{A}{r}\right) \cdot r \, dr \\
p2 \leftarrow \int_{rc}^R \psi(m, x0m \cdot r) \cdot \left(\frac{\pi}{4} - \text{asin}\left(\frac{A - \frac{B}{2}}{\sqrt{2} \cdot r}\right)\right) \cdot r \, dr \\
\frac{4}{\pi} \cdot (p1 + p2)
\end{array} \right.
\end{array} \right. \\
\end{array}
\end{aligned}$$

pickup plates aligned along  $\varphi=0$ , integral uses cosine

both opposite plates -- extra factor of 2 if even, zero if odd.

$m=0$  equivalent to  $\sin(mx)/m=x$  in the integrand

drive plate parameters

$$\underline{\underline{A}} := \frac{0.200 \cdot 0.0254}{2 \cdot a}$$

$$\underline{\underline{B}} := 2 \cdot A$$

drive plate aligned along the y axis ( $\varphi = \pi/2$ )

$$\begin{aligned}
D(m) := & \left| \begin{array}{l}
x0m \leftarrow xm_m \\
rc \leftarrow \sqrt{A^2 + \left(\frac{B}{2}\right)^2} \\
\text{if } m > 0 \\
\quad \left| \begin{array}{l}
p1 \leftarrow \int_A^{rc} \psi(m, x0m \cdot r) \cdot \sin\left(m \cdot \text{acos}\left(\frac{A}{r}\right)\right) \cdot r \, dr \\
p2 \leftarrow \int_{rc}^R \psi(m, x0m \cdot r) \cdot \sin\left[m \cdot \left(\frac{\pi}{4} - \text{asin}\left(\frac{A - \frac{B}{2}}{\sqrt{2} \cdot r}\right)\right)\right] \cdot r \, dr \\
\frac{2 \cdot \exp\left(i \cdot m \cdot \frac{\pi}{2}\right)}{m \cdot \pi} \cdot (p1 + p2)
\end{array} \right. \\
\text{otherwise} \\
\quad \left| \begin{array}{l}
p1 \leftarrow \int_A^{rc} \psi(m, x0m \cdot r) \cdot \text{acos}\left(\frac{A}{r}\right) \cdot r \, dr \\
p2 \leftarrow \int_{rc}^R \psi(m, x0m \cdot r) \cdot \left(\frac{\pi}{4} - \text{asin}\left(\frac{A - \frac{B}{2}}{\sqrt{2} \cdot r}\right)\right) \cdot r \, dr \\
\frac{2}{\pi} \cdot (p1 + p2)
\end{array} \right.
\end{array} \right.
\end{aligned}$$

one plate only (D1) with the y-axis phase factor applied to cosine integral

m=0 equivalent to sin(mx)/m=x in the integrand with no phase factor

### Gap Calibration

The transducer coupling requires knowledge of the electrode geometry. The area is assumed to be known, but the gap is deduced from the experimental capacitance. Experimentally, the LC oscillation frequency of the pickup detector (the TDO) is the accessible quantity. To get back to the pickup capacitance, we need to know the inductance (L) of the LC circuit and the amount of any stray capacitance NOT included as part of the gap. The inductance and physical areas are assumed to be the same as their room temperature determinations.

We thus have two unknown quantities that need to be experimentally determined: the gap and the stray capacitance. These unknowns are determined by the (1) the empty cell TDO frequency, and (2) the shift in the TDO frequency when only the gap is filled with liquid.

cell TDO inductance  $L := 0.365 \cdot 10^{-6}$

empty TDO frequency  $f_{tdo} := 74513695$

O017 - O022 Helium Fill

shift when filled  $\Delta f_{\text{fill}} := 1506527$

He or N2 dielectric  $\epsilon_m := \epsilon_{\text{He}}$

ratio of pickup capacitance to total  $cpr := \frac{\left(\frac{f_{\text{tdo}}}{f_{\text{tdo}} - \Delta f_{\text{fill}}}\right)^2 - 1}{\epsilon - 1}$   $cpr = 0.758$

capacitances  $C_{\text{tot}} := \frac{1}{(2 \cdot \pi \cdot f_{\text{tdo}})^2 \cdot L}$   $C_p := cpr \cdot C_{\text{tot}}$   $C_s := C_{\text{tot}} - C_p$

pickup plates  $C_p = 9.476 \times 10^{-12}$  other stray capacitances  $C_s = 3.023 \times 10^{-12}$

capacitor gap  $gap := \frac{\epsilon_0 \cdot A_p}{C_p}$   $gap = 2.401 \times 10^{-5}$   $f_{\text{tdo}} := 74093695$

Cell Properties

Mode Details

Only the mechanical (0,0) and (2,0) modes are valid!

(2,0)  $m := 2$   $f_{\text{drive}} := 17860$  Hz  $Q := 29400$

mode of interest  $xmn := x_{m_m}$   $xmn = 2.465$

Assumed drive voltage corrected for: (1) 3X box factor of 3; and (2) 36 kHz amplitude and phase shift of the transformer, if used. See O72 for this correction. This phase factor appropriately doubles through the squaring conversion below.

$$A_{3x} := 3.045 \quad \varphi_{3x} := \frac{73.6}{180} \cdot \pi$$

$$V_d := 0.5 \cdot A_{3x} \cdot e^{i \cdot \varphi_{3x}}$$

In the following, the drive frequency  $f_{\text{drive}}$ , is the actual frequency of the drive oscillator. The applied force

will be at twice this frequency due to the electrostatic frequency doubling..

fundamental resonance relations  $\omega = h \cdot \left(\frac{x_0}{a}\right)^2 \cdot \sqrt{\frac{Y}{12 \cdot \rho \cdot (1 - \nu^2)}} \quad k = \frac{x_{mn}}{a}$

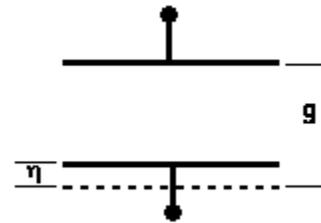
drive and pickup overlaps (two drives)  $\underline{D} := 2 \cdot D(m) \quad \underline{P} := P(m)$

$D = -0.271 \quad P = 0.225$

The transducer coupling is found by calculating the DC response of the mode to a fixed voltage, then assuming a simple harmonic response to the AC drive.

The mode DC response to the drive is to be interpreted as the displacement of the mode, taken as a rigid motion, in response to the static electric field in the capacitor. The displacement is found by minimizing the combined total elastic energy and the electrostatic energy.

The capacitance change of an electrode plate element with a plate change  $h$  is found from the series combination of the vacuum and film section elements of the electrode. This is then integrated for the parallel capacitance of all the elements. This will be valid if the wavelength of the mode is much greater than the gap.



Note that a positive displacement  $\eta$  is defined to reduce the electrode gap.

plate shape:  $\eta(r, \phi) = \eta \cdot \psi(r, \phi)$

capacitance elements  $dC = \frac{\epsilon_0 \cdot dA}{g - \eta} - \frac{\epsilon_0 \cdot dA}{g} \quad dC = \frac{\epsilon_0 \cdot dA}{g} \cdot \frac{\eta}{g}$

The total change is the integral over drive or pickup, expressed in terms of  $D$  or  $P$  defined earlier

$$\Delta C = \frac{\epsilon_0}{g^2} \cdot \eta \cdot \int \psi^2 dA = \frac{\epsilon \cdot \pi \cdot a^2}{g^2} \cdot \eta \cdot \begin{pmatrix} D \\ P \end{pmatrix}$$

The electrostatic energy change of the drive plate includes the work done by the voltage source at  $V_d$ .

$$\Delta U_{\text{elec}} = \frac{1}{2} \cdot \Delta C \cdot V_d^2 - V_d \cdot \Delta Q = -\frac{1}{2} \cdot \Delta C \cdot V_d^2 = -\frac{\pi \cdot a^2 \cdot \epsilon_0 \cdot V_d^2}{2 \cdot g^2} \cdot \eta \cdot D$$

The elastic energy is the same as the kinetic energy, which from the "Disk Displacements" section is...

$$\Delta U_{\text{elastic}} = \frac{1}{2} \cdot \kappa \cdot \eta^2 \quad \kappa = \frac{\pi \cdot Y \cdot a^2 \cdot h^3 \cdot k^4}{12 \cdot (1 - \nu^2)} = \pi \cdot a^2 \cdot h \cdot \rho \cdot \omega^2$$

$0 = \frac{d}{d\eta} (\Delta U_{\text{elec}} + \Delta U_{\text{elastic}})$  determines the equilibrium plate displacement in the static field,  $\eta_{\text{DC}}$

$$-\frac{\pi \cdot a^2 \cdot \epsilon_0 \cdot V_d^2}{2 \cdot g^2} \cdot D + \kappa \cdot \eta = 0 \quad \omega := 2 \cdot \pi \cdot 35720$$

use  $k = \frac{x_{\text{mn}}}{a}$   $\kappa_{\text{th}} := \frac{\pi \cdot Y \cdot h^3 \cdot x_{\text{mn}}^4}{12 \cdot (1 - \nu^2) \cdot a^2}$   $\kappa_{\text{ex}} := \pi \cdot a^2 \cdot h \cdot \rho \cdot \omega^2$

$$\kappa_{\text{th}} = 1.249 \times 10^7 \quad \kappa_{\text{ex}} = 1.33 \times 10^7$$

choose which  $\kappa$  to use  $\kappa := \frac{\kappa_{\text{ex}} + \kappa_{\text{th}}}{2}$

$$\eta_{\text{DC}} := \frac{1}{2} \cdot \frac{\epsilon_0 \cdot \pi \cdot a^2 \cdot V_d^2}{\text{gap}^2 \cdot \kappa} \cdot D \quad \eta_{\text{DC}} = 4.18 \times 10^{-14} - 2.694i \times 10^{-14} \quad \text{m}$$

The AC response has 1/2 the amplitude at twice the frequency of the drive, from  $\cos(\theta)^2 = \frac{1 + \cos(2\theta)}{2}$ , and the frequency dependence of a mass and spring:

$$\eta = \frac{\frac{1}{2} \cdot \eta_{\text{DC}}}{1 - \left(\frac{\omega}{\omega_0}\right)^2 - \frac{i}{Q} \cdot \frac{\omega}{\omega_0}} \quad e^{-i \cdot \omega \cdot t} \quad \text{convention}$$

On resonance,  $\eta = \frac{i}{2} \cdot \eta_{\text{DC}} \cdot Q$

thickness oscillations on resonance  $\eta_{\text{mn}} := \frac{i \cdot Q}{4} \cdot \frac{\epsilon_0 \cdot \pi \cdot a^2 \cdot V_d^2}{\text{gap}^2 \cdot \kappa} \cdot D$

disk mode amplitude  $\eta_{mn} = 3.96 \times 10^{-10} + 6.145i \times 10^{-10}$  m

TDO frequency modulation comes from the capacitance changes with the disk amplitude  $\eta$

$$\frac{\Delta f}{f} = -\frac{1}{2} \cdot \frac{\Delta C_p}{C} \quad \text{with} \quad \Delta C_p = P \cdot \frac{\epsilon_0 \cdot \pi \cdot a^2}{\text{gap}} \cdot \frac{\eta}{\text{gap}}$$

$$\frac{\Delta f}{\eta} = -\frac{1}{2} \cdot \frac{f_{tdo}}{\text{gap}} \cdot \frac{f_{tdo}}{C_{tot}} \cdot \frac{\Delta C_p}{\eta} = -\frac{1}{2} \cdot \frac{f_{tdo}}{\text{gap}} \cdot \frac{C_p}{C_{tot}} \cdot \frac{\pi \cdot a^2 \cdot P}{A_p}$$

This is the mode sensitivity of the LC oscillator. The response to  $\eta$  is cut down by (1) the moderating effect of the stray capacitance and (2) the shape of the mode within the pickup region.

mode sensitivity  $df/d\eta := -\frac{1}{2} \cdot \frac{f_{tdo} \cdot cpr \cdot \pi \cdot a^2 \cdot P}{\text{gap} \cdot A_p}$   $df/d\eta \cdot 10^{-9} = -1.361 \times 10^3$   $\frac{\text{Hz}}{\text{nm}}$

Here's the theoretical frequency modulation of the TDO for all of the conditions specified previously...

$$f_{mn} := \eta_{mn} \cdot df/d\eta \quad f_{mn} = -538.99 - 836.348i$$

Here is the complete expression in several forms  $\eta := 10^{-9}$   $P = 0.225$   $D = -0.271$

most reduced  $f_{mn} := -\frac{i \cdot \pi^2}{8} \cdot Q \cdot D \cdot P \cdot cpr \cdot \frac{\epsilon_0 \cdot a^4 \cdot V_d^2 \cdot f_{tdo}}{\kappa \cdot \text{gap}^3 \cdot A_p}$

driven mass-and-spring

$$f_{mn} = \left( -\frac{1}{2} \cdot \frac{f_{tdo}}{\text{gap}} \right) \cdot (\text{res}) \cdot \left( \frac{\text{electric}_\kappa}{\text{elastic}_\kappa} \right) \cdot cpr \cdot (\text{pickup\_overlap})$$

full **spring constant** expression

$$f_{mn} := \left( -\frac{1}{2} \cdot \frac{f_{tdo}}{\text{gap}} \right) \cdot i \cdot Q \cdot \left( \frac{1}{2} \cdot \frac{\epsilon_0 \cdot \pi \cdot a^2 \cdot D \cdot V_d^2}{\kappa \cdot \text{gap}^2} \cdot \frac{V_d^2}{2} \right) \cdot cpr \cdot \frac{\pi \cdot a^2 \cdot P}{A_p}$$

$$f_{mn} = -538.99 - 836.348i$$

Note: All of the factors of 2 are associated with particular terms; The D factor applies to the full mechanical area; The P factor only applies relative to the pickup electrode; The cpr factor accounts for non-pickup capacitance.

full **theoretical elasticity**  
expression

$$f_{y_{mn}} := i \cdot \left( -\frac{1}{2} \cdot f_{ido} \cdot cpr \cdot \frac{P \cdot \pi \cdot a^2}{A_p} \cdot \frac{\eta}{gap} \right) \cdot \frac{\frac{1}{2} \cdot \frac{\epsilon_0 \cdot D \cdot \pi \cdot a^2}{gap} \cdot \left( \frac{V_d}{2} \right)^2 \cdot \frac{\eta}{gap}}{\frac{1}{Q} \cdot \frac{\pi}{24} \cdot \frac{Y \cdot h^3 \cdot x_{mn}^4 \cdot \eta^2}{(1 - \nu^2) \cdot a^2}}$$

$$f_{y_{mn}} = -556.635 - 863.729i$$

full **experimental elasticity**  
expression

$$f_{\omega_{mn}} := i \cdot \left( -\frac{1}{2} \cdot f_{ido} \cdot cpr \cdot \frac{P \cdot \pi \cdot a^2}{A_p} \cdot \frac{\eta}{gap} \right) \cdot \frac{\frac{1}{2} \cdot \frac{\epsilon_0 \cdot D \cdot \pi \cdot a^2}{gap} \cdot \left( \frac{V_d}{2} \right)^2 \cdot \frac{\eta}{gap}}{\frac{1}{Q} \cdot \frac{\pi \cdot a^2 \cdot h \cdot \rho \cdot \omega^2}{2} \cdot \eta^2}$$

$$f_{\omega_{mn}} = -522.428 - 810.65i$$

Mode Details

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Fit Prediction

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## Prediction for the Resonance Fits

At this point, we switch to the  $e^{i \cdot \omega \cdot t}$  convention since both the lock-in and the fitscan program use this sign for i.

$$f_{\omega_{mn}} := \overline{f_{mn}}$$

The "Phase-Locked-Loop" (PLL) detection system changes frequency shifts to voltage with some gain,  $G_{PLL}$ . This gain is calibrated by measuring the lockin response to a given modulation of the reference oscillator. The PLLCAL program reports this response (at  $2 \cdot f_{drive}$ ), which is opposite in sign to the response to the signal frequency shifts.

$$G_{PLL} = \frac{\delta V_{in}}{\delta f} \quad PLLCAL = \frac{\delta V_{lockin1}}{\Delta f_{ref}} = -\frac{\Delta V_{lockin1}}{\Delta f} \quad \frac{\delta V_{lockin1}}{\delta V_{in}} = \frac{1}{\sqrt{2}} \quad (1f \text{ mode})$$

$$\text{so, with O76A scan} \quad PLLCAL := (10.25 + 1.3i) \cdot 10^{-6} \quad G_{PLL} := -\sqrt{2} \cdot PLLCAL$$

The data, however, is measured with the lock-in referenced to the drive frequency, which gets frequency doubled by the  $V^2$  electrostatic drive. As measured in "2f" mode. It is also reported as an RMS voltage with an additional factor of  $i$ . (See O71 for a confirmation of this).

$$\frac{\delta V_{\text{lockin}2}}{\delta V_{\text{in}}} = \frac{i}{\sqrt{2}} \quad 2f \text{ mode}$$

The lock-in signal on resonance is then  $V_{\text{lockin}} = f_{\text{mn}} \cdot G_{\text{PLL}} \cdot \frac{\delta V_{\text{lockin}2}}{\delta V_{\text{in}}} = -i \cdot f_{\text{mn}} \cdot \text{PLL CAL}$

The FitScan program takes the lock-in readings and fits to the form with Q factored into the amplitude:

$$V_{\text{lockin}}(f) = \frac{A_{\text{fit}}}{2 \cdot Q \cdot \left(1 - \frac{f}{f_0}\right) + i} + x_0 + i \cdot y_0 \quad A_{\text{fit}} = \text{complex}$$

So the resonant lock-in response would be  $V_{\text{lockin}} = \frac{A_{\text{fit}}}{i}$

**predicted fit parameters**

$$A_{\text{th}} := f_{\text{mn}} \cdot \text{PLL CAL}$$

$$A_{\text{th}} = -6.612 \times 10^{-3} + 7.872i \times 10^{-3}$$

**experimental fit**

O76A

$$m = 2 \quad f_0 = 17860 \quad Q = 29400 \quad A_{\text{fit}} := 0.00414 \cdot e^{i \cdot 0.0081}$$

**sensitivity ratio**

$$\text{SR} = \frac{\text{experimental}}{\text{theoretical}} \quad \text{SR} := \frac{A_{\text{fit}}}{A_{\text{th}}}$$

$$\text{SR} = -0.257 - 0.31i \quad \left( e^{i \cdot \omega \cdot t} \right)$$

$$\overline{\text{SR}} = -0.257 + 0.31i \quad \left( e^{-i \cdot \omega \cdot t} \right)$$

## Summary Results

old analysis (062?) with previous versions of the analysis

$$m = 2 \quad f = 17860 \quad V_d = 1.5 \quad Q = 34000 \quad \text{PLL} = -3.55 + 0.25i \quad \frac{\mu\text{V}}{\text{Hz}}$$

$$x_{mn} = 2.465 \quad D = -0.271 \quad P = 0.225$$

$$\eta_{dc} = -4.985 \cdot 10^{-14} \text{ m} \quad df d\eta \cdot 10^{-9} = -1369 \quad \frac{\text{Hz}}{\text{nm}}$$

$$\eta_{mn} = -8.475i \times 10^{-10} \text{ m} \quad f_{mn} = 1.16i \times 10^3 \text{ Hz}$$

$$A_{mn} = -290.001 - 4.118i \times 10^3 \quad \mu\text{V}$$

$$m = 0 \quad f = 15580 \quad V_d = 1.5 \quad Q = 26000 \quad \text{PLL} = -3.25 + 0.1i \quad \frac{\mu\text{V}}{\text{Hz}}$$

$$x_{mn} = 2.177 \quad D = 0.309 \quad P = 0.206$$

$$\eta_{dc} = 9.305 \cdot 10^{-14} \text{ m} \quad df d\eta \cdot 10^{-9} = -1252 \quad \frac{\text{Hz}}{\text{nm}}$$

$$\eta_{mn} = 1.21i \times 10^{-9} \text{ m} \quad f_{mn} = -1.514i \times 10^3 \text{ Hz}$$

$$A_{mn} = 151.407 + 4.921i \times 10^3 \quad \mu\text{V}$$

Fit Prediction

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Revision History

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Summer 2010

Modified the third sound analysis program, "stimulated condensation resonator.xmcd", into this document and analyzed O062 (B15P062) mechanical scans.

09/08/10

Carefully added or modified:the 3x box phase shift, TDO-REF sign, on-resonance i, and 2f mode i.

09/09/10

Changed the focus to predict (1) the mode amplitudes  $f_{mn}$  and  $\eta_{mn}$ , then (2) the fit program parameters. The physics from the detection. Older version saved as C:\Fred\Physics\FILMS\third sound\resonators\stimulated\_condensation\stimulated\_condensation\_disk\_v1.xmcd

 Revision History

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