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# On Third Sound Amplification by Stimulated Condensation

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*We describe the process by which condensing $^4$He vapor atoms are expected to contribute coherently to the macroscopic superfluid motion of a third sound resonance. This phenomenon is similar to the experimentally observed thickening of a film in the fixed velocity state of a persistent current, though with an interesting and subtle difference. As before, the condensing atoms contribute to the kinetic energy of the macroscopic state by the coherent normal-to-superfluid conversion. In the present case, however, the macroscopic state is an oscillatory third sound mode. This allows the energy to remain in the third sound mode even if superfluid is removed by film flow in order to keep the film thickness constant. Thus, a continuous process of third sound amplification by stimulated condensation should accompany a condensing vapor flux under the right conditions.*

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## 1. INTRODUCTION

That the superfluid state in a film with no flow grows as atoms are condensed into it from the vapor is not surprising. Atoms condense into the film, increasing its mass (and thickness), and as long as the temperature of the film is maintained below the superfluid transition temperature, the equilibrium superfluid state is preserved. It is at first surprising that the velocity state of a persistent current is also preserved as the film thickness grows due to vapor condensation[1]. When considered in conjunction with the presence of a macroscopic quantum state, it becomes intuitive. The flow speed is built into the macroscopic phase structure of the superfluid, and a microscopic vapor condensation event would be extremely unlikely to influence it. Changing the flow velocity would require a change in the phase count around macroscopic paths involving macroscopic quantities of



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superfluid mass.

This paper will investigate the consequences of film growth through vapor condensation as it applies to macroscopic third sound oscillations. We will begin by reviewing the details of the process in persistent currents. We then consider the nature of third sound as a macroscopic quantum state and the corresponding condensation of particles into third sound modes. Finally, extraction of mass from the mode by film flow is discussed where it is will be seen that the mode energy gained during the condensation is preserved in the third sound as the film is extracted. The result is an amplification of the mode energy at a fixed film thickness if the condensing vapor is continuously recycled via film flow.

## 2. PERSISTENT CURRENTS

The quantum phase structure of a superfluid associated with flow is essentially a de Broglie wave state applied to $^4$He atoms in which all of the mass participates. A spatially dependent phase $\Phi(\mathrm{r})$ can be associated with the classical velocity potential such that

$$\vec{v} = \frac{\eta}{m_4}\nabla\Phi \quad with \quad \oint \vec{v}\cdot d\vec{\lambda} = \frac{2\pi\hbar}{m_4}N \tag{1}$$

Persistent currents are thus characterized by a static wavefunction phase that varies in space along the streamlines of the fluid flow and quantizes the velocity field circulation. Changes in the topology of the lines require that vortex pairs be formed and moved apart, or moved together and annihilated, and can only happen with macroscopic influences at low temperatures. Condensing atoms would not be able to cause such changes since they are microscopic events.

The details of a condensation event are understood in terms of thermal excitations[2]. In principle, the impinging atom state can be expressed as a superposition of elementary excitations in the film, and it is these excitations that conserve energy and momentum for the process. In this picture, it is the dominance of the ground state (within which the thermal excitations are considered) that guarantees that the fate of a condensing atom's mass is assimilation into superfluid. The excitations are ultimately thermalized.

In a moving film, the "ground state" within which the thermal excitations are considered is now the state of persistent current, along with its associated phase structure. This is the appropriate base state in which to place excitation since it is the state the film would be left with as the temperature were reduced to zero.

Atoms condensing into a moving film are thus expressed in terms of slightly different sets of excitations - ones with a zero occupation state that has the phase periodicity of the persistent current. This periodicity, from the phase relations (1), would have a wavelength of approximately $50\mu$m for a 20 cm/s film flow and is significantly longer than the wavelength of the typical excitation produced by condensation events.

If the film is a persistent current adsorbed onto the surface of a ring, both the angular momentum and the energy of the superfluid are increased proportionally to the increase in the mass of the film. Of course, angular momentum and energy are conserved, with the excitations deposited in the film assuring a proper accounting of the former. The angular momentum and energy changes experienced by the condensing vapor atoms as they assimilate into the moving film will ultimately be transmitted to the substrate as the excitations thermalize.

The condensation thus converts heat into mechanical energy. This energy shows up as a thicker film moving with the constant flow speed. Unfortunately, it is not possible to remove the added atoms by film flow in some continuous process that leaves the energy behind as an increased circulation speed. As atoms are removed from the flow region, their kinetic energy is removed with them: any extraction path away from the flow is subject to the same potential flow constraints that give rise to the constant circulation condition (1), and would preserve the velocity.

## 3. THIRD SOUND

Now consider the third sound mode in a film. The flow field is oscillatory in nature and also thermally active. The mode is consequently not persistent to the same extent as a DC flow. There are three aspects of this situation to keep in mind:

i) Third sound has a wavelength $2\pi/k$ on the order of a centimeter and an associated velocity field that extends over this wavelength. As with the persistent current, the velocity field has a quantum phase structure which is described by the phase relations of Eqs. (1). Combined with the equations of motion for third sound, the phase structure has the form

$$\Phi = \frac{m_4 c_3}{\hbar k} \frac{\eta}{h} \tag{2}$$

where $\eta$ is displacement of the wave surface from the static thickness $h$ (taken to be 3 nm for numerical values presented), and $c_3$ is the third sound speed.

ii) In thermal equilibrium, third sound is considered to be a part of the elementary excitation spectrum and is thermally populated to a harmonic

oscillator level given by the Planck distribution, which at $T = 0.1K$, is about $n = 10^6$. If the amplitude at this excitation level is applied to the phase relation (2) above, the derived thermal phase oscillations are small and vary only slightly over the wavelength of the third sound. This can be considered as a microscopic amplitude regime.

iii) Third sound can be classically driven to quite high amplitudes. For an amplitude with $\eta/h = 0.01$, the equivalent harmonic oscillator excitation level would be approximately $n = 10^{18}$. Here, the quantum phase structure would have a wavelength much smaller than the third sound wavelength, on the order of $50\mu$m. Although the wave is quite classical, with an extremely high level of excitation, the instantaneous flow field is well described by a quantum phase structure, similar to a persistent current.

Condensation of vapor into a film with a third sound wave present thus has two distinct regimes - microscopic flow or macroscopic flow. In the microscopic case, the third sound mode is mearly an independent collective excitation and incoherently participates in the condensation process only to the extent that it is needed in the superposition of excitations describing the condensing atom.

The macroscopic flow case is different. The quantum phase structure of the classical wave field requires the same subtle adjustments in the excitations created that occur in the persistent current case. The created excitations (as well as the thermally populated excitations) are referenced to the macroscopic quantum phase structure of the third sound mode for their zero occupation base state - the mode being a well defined state as the temperature of film is reduced to zero.

Condensing atoms are thus assimilated into the quantum phase structure of the classical third sound wave flow-field, preserving the local character of the phase structure, the local character being the film velocity. Within the classical velocity field of the third sound wave, microscopic condensation events are indistinguishable from those in a persistent current, both of which stimulate atoms to condense coherently into the phase structure due to an overwhelming occupation of the Bose state.

## 4. ACOUSTIC AMPLIFICATION

We will now restrict the discussion to third sound modes of well defined non-zero angular momentum. These are the rotational modes of a circular resonator with azimuthal eigenvalue $m > 0$, for example. Just as with the persistent current on a ring, the energy increases proportional to the thickness change since the condensing atoms pick up the kinetic energy associated

with the local flow. The constant of proportionality, $\alpha$, depends on the coincidence of the kinetic energy distribution within the mode and the vapor flux distribution. For harmonic waves with a uniform vapor flux, $\alpha = \frac{1}{2}$. The effect on the oscillator excitation level and the angular momentum is subtle. They both scale as $E/\omega$, but the frequency $\omega$ of the mode decreases with increasing film thickness. The harmonic oscillator level n and the angular momentum L are related to the mode energy by

$$n = \frac{E}{\hbar\omega} \qquad L = \frac{mE}{\omega} \tag{3}$$

and these change with thickness such that $dE/dh = \alpha E/h$. Using this, and $\omega = c_3 k$ with $c_3 = \sqrt{hU_h}$, one finds that the angular momentum and energy excitation level change with condensed atoms such that

$$\frac{dL}{dh} = \frac{\alpha}{2}\frac{L}{h}\left(1 - \frac{hU_{hh}}{U_h}\right) \quad and \quad n = \frac{L}{m\hbar} \tag{4}$$

where $U$, $U_h$, etc are the attractive potential function of the substrate and its derivatives. Note that $U_h > 0$ and $U_{hh} < 0$ for all attractive substrates. For third sound in a van der Waals potential, $dL/dh = \frac{5}{2}aL/h$. The angular momentum no longer grows proportional to the added mass, as with persistent currents, reflecting the acoustic character of the mode.

Vapor condensation can add energy and angular momentum to either a persistent current flow field or a third sound wave flow field. The difference in the rate of growth of the angular momentum vs. energy illustrates an interesting feature of the acoustic mode case. The acoustic mode holds the energy and momentum in a distinctly different way than a persistent current. First of all, only half of the energy in the acoustic mode is kinetic, where it is all kinetic in the persistent current. Secondly, the angular momentum of a persistent current is derived from extended circulation of the superfluid, as in Eq. (1). Acoustic angular momentum has its root in the small oscillatory orbital motion of fluid elements within a field of zero circulation.

We saw that a persistent current would loose its energy and momentum if mass were drained away by film flow, preserving the circulation. This is not the case for the acoustic mode, where the angular momentum is contained in the localized fluid oscillations. If film mass is removed from the resonator in an axisymmetric, radial DC flow, the angular momentum of the oscillatory acoustic mode will be preserved[3]. The energy actually increases, receiving the work done against the substrate potential as film is pulled away, but the excitation level, being proportional to $L$, is constant.

With this decoupling of the acoustic mode from the DC film flow, a continuous input of energy, through vapor accommodation and simultaneous

mass extraction, is possible. With the gain in harmonic oscillator quantum number given by Eq. (4), the energy gain is

$$\frac{dE}{dh} = \frac{\alpha}{2}\frac{E}{h}(1 - \frac{hU_{hh}}{U_h}) \tag{5}$$

where $h$ can now be interpreted as the equivalent film height of the mass passed into the mode from the vapor, and out of the mode through film flow. The energy gain is proportional to the energy stored in the mode, and will have the effect of reducing the dissipation of the third sound mode, or if large enough, resulting in a self oscillation. The condition for this self oscillation can be written in terms of the free mode quality factor $Q$:

$$\frac{dE}{dh}\frac{dh}{dt} > \frac{\omega E}{Q} \quad or \quad \frac{dh}{dt} > \frac{2}{5}\frac{\omega h}{\alpha Q} \quad for \;\; U(h) \propto h^{-3} \tag{6}$$

This indicates that the condition for self oscillation requires that approximately all of the atoms in the film be replaced within the natural free decay time of the mode resonance.

## 5. CONCLUSIONS

A third sound resonance should be amplified by exposing the film to a flux of vapor atoms and removing the atoms via film flow out of the center and the accompanying thermal energy out through the substrate. The resonance state must be one of a definite, non-zero angular momentum. Self-sustaining oscillations will be possible if competing attenuation is small enough. We are currently working on an apparatus to test these conclusions. Our third sound resonator is specially designed to meet the required conditions of an open geometry, high Q, and sufficient thermal anchoring[4].

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