VAPOR EQUILIBRIUM FOR A HELIUM FILM (MKS)

Start with the saturated vapor pressure of liquid helium, reduce it to an unsaturated vapor within the van der Waals potential at a height h, then find the film thicness in a box with a fixed number of atoms

Saturated vapor pressure from a fit to table...

$$P_s(T) := \exp\left[\frac{4.24846}{\frac{1}{T} + .524764} + 11.8971 - 7.80921\cdot\left(\frac{1}{T} + .524764\right)\right] \quad \text{(Kelvin, Pascals)}$$

Pressure in equillibrium with a film, thickness h is reduced by the Boltzman factor...

film constants... $T_v := 39$ $\quad h_1 := 3.578\cdot 10^{-10}$ $\quad k := 1.3805\cdot 10^{-23}$ $\quad h_r := 41.7\cdot h_1$

$$P(T,h) := P_s(T)\cdot\exp\left[-\frac{T_v}{T}\cdot\left(\frac{h_1}{h}\right)^3\cdot\frac{1}{\left(1 + \frac{h}{h_r}\right)}\right]$$

This relates the three quantities, P, T, and h, so any two determine the third. The area and volume of the container determine where the helium is at any temperature. Ideal gas behavior is assumed in the vapor.

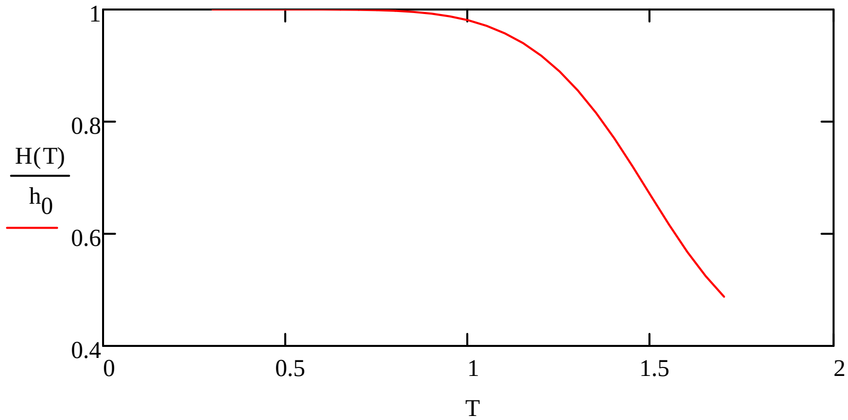
Area $A := 10$ $\quad$ Volume $V := 10\cdot 10^{-6}$ $\quad$ T=0 thickness $h_0 := 2.5\cdot 10^{-9}$

constant number of particles $\quad \frac{P(T,h)\cdot V}{k\cdot T} + \frac{h\cdot A}{h_1^{\,3}} = \frac{h_0\cdot A}{h_1^{\,3}}$

This can be solved numerically for the thickness to show how the film eveporates as temperature is raised...

$$x := .9\cdot\frac{h_0}{h_1} \qquad vok := \frac{V\cdot h_1^{\,3}}{k\cdot h_0\cdot A} \qquad H(T) := h_0\cdot\mathrm{root}\left(\frac{P(T, x\cdot h_0)}{T}\cdot vok + x - 1, x\right)$$

$T := .30, .35 .. 1.7$



Note that, for our third sound chamber containg a surface reservoir, the evaporative thinning turns on well above 0.5 K.